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N-LOOP VLF SUPERDIRECTIVE ARRAYS

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FOREWORD

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ABSTRACT

Superdirectivity may be achieved with short VLF loop arrays because the beam width depends only upon the number of loops and not upon the length of the array. In addition, the usual factors limiting superdirectivity are not as prevalent in these arrays because of the decoupling between VLF loops.

Expressions are derived for the beam width, effective height, reception pattern, amplitude and position of the back lobes, and the effects of voltage phase and amplitude differences between loops. These equations describe short arrays of any number of loops. The most serious limitation concerning the directivity of superdirective loop arrays is caused by the voltage phase and amplitude differences between loops. These differences between adjacent loops combine to obscure the nulls and deteriorate the reception pattern.

CONTENTS

INTRODUCTION

The superdirective antenna of infinitesimally small size, with its possibility of infinite gain, has been discussed by several authors (Ref. 1-6). These authors have pointed out that such arrays are impractical because of basic limitations such as narrow bandwidth, high losses, and critical tolerances in individual loop voltages. Nevertheless, moderate superdirectivity has been achieved (Ref. 7, 8) with practical arrays. It has been shown (Ref. 8) that superdirectivity can be obtained from radiators that are decoupled from one another. The elements of receiving arrays at VLF can very easily be decoupled since they are so small compared with the wavelength.

At very low frequencies, it is very difficult to obtain highly directive antenna patterns. Large tracts of land are needed because of the long wavelengths involved. However, it appears that arrays can be greatly reduced in size by using the principle of superdirectivity. Considerable directivity can be achieved with a superdirective loop array that is only a small fraction of a wavelength in size. These superdirective receiving antennas are possible at VLF because the decoupling between loops (Ref. 9) makes the usual limiting factors, such as narrow bandwidth and high losses, extremely small. The critical tolerance in individual loop voltages is the factor that limits the number of loops (n) that can be used, which limits the directivity.

The characteristics and performance of short two- and three-loop arrays have been discussed previously (Ref. 9 and 10). This report will expand the subject to include the radiation pattern characteristics, the effective height, and the effect of phase and amplitude errors on the short n-loop superdirective array.

RADIATION PATTERN CHARACTERISTICS

In this section, the important characteristics of the n-loop array pattern are deduced from corresponding equations of the two- and three-loop arrays. Equations for the positions of the side lobes and side nulls, the beam width, and the ratios of side lobe and back lobe to front lobe are presented.

The horizontal pattern of the horizontal array with the planes of the loops oriented in a vertical plane (Ref. 9) is shown in Figure 1. This pattern is described by equations (1), (2), and (3). For the two-loop array

$$|E_\varphi| = \cos \varphi \left[\frac{2\pi D}{\lambda} (\cos \varphi - \cos \varphi_0) \right] \tag{1}$$

and for the three-loop array

$$|E_\varphi| = \cos \varphi \left[\frac{2\pi D}{\lambda} (\cos \varphi - \cos \varphi_0) \right]^2 \tag{2}$$

where

E_φ = relative voltage received from direction φ compared with the voltage from one loop

φ = angle of received signal in the horizontal plane measured from the plane of loops

D = distance between loops

λ = free-space wavelength

In these equations, $D \ll \lambda$, and there is a phase difference of $\delta - \pi$ between identical adjacent loops, where δ is the delay between adjacent loops. By mathematical induction, the pattern for the n-loop array is described as

$$|E_\varphi| = \cos \varphi \left[\frac{2\pi D}{\lambda} (\cos \varphi - \cos \varphi_0) \right]^{n-1} \tag{3}$$

Equation (3) is expressed in terms of (a) distance between loops in wavelengths, D/λ, and (b) the null position, φ_0, that is located between the back lobe and the side lobes (see Figure 1). For the n-loop array, the null position depends upon the delay between adjacent loops and the free-space propagation time between loops, as expressed by

$$\varphi_0 = \text{arc cos} \frac{\delta}{D/v_0} \tag{4}$$

where v_0 is the velocity of light. The null position may be moved about the back half of the pattern to reject unwanted signals by varying the delay, δ, between adjacent loops.
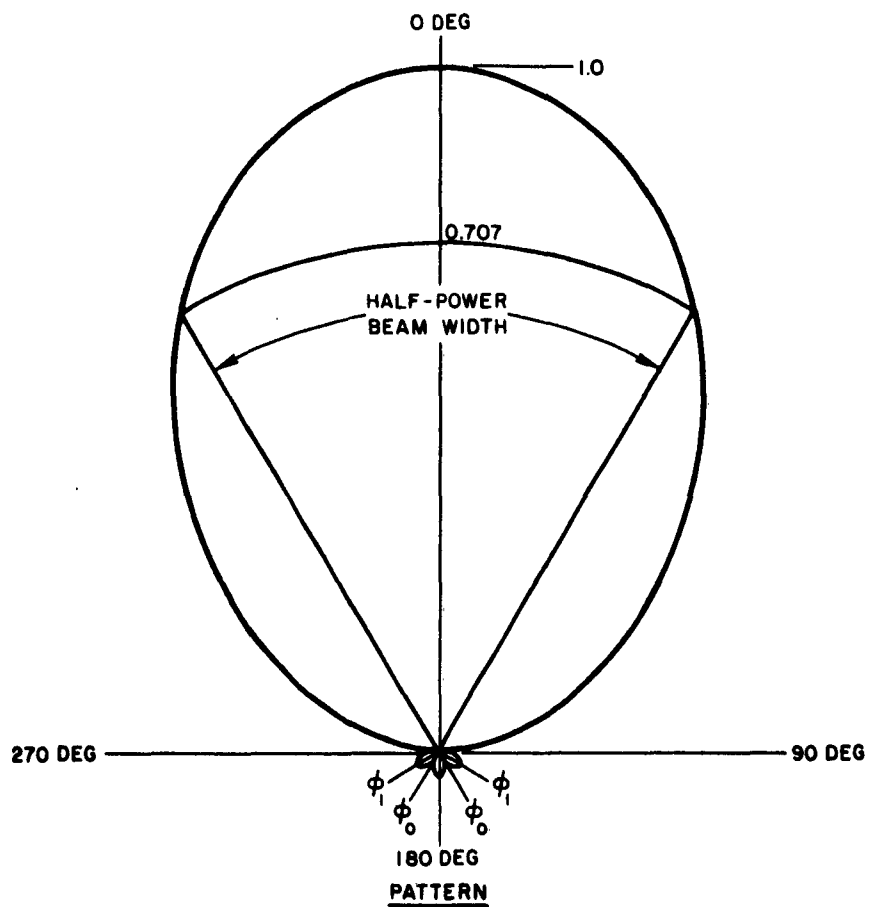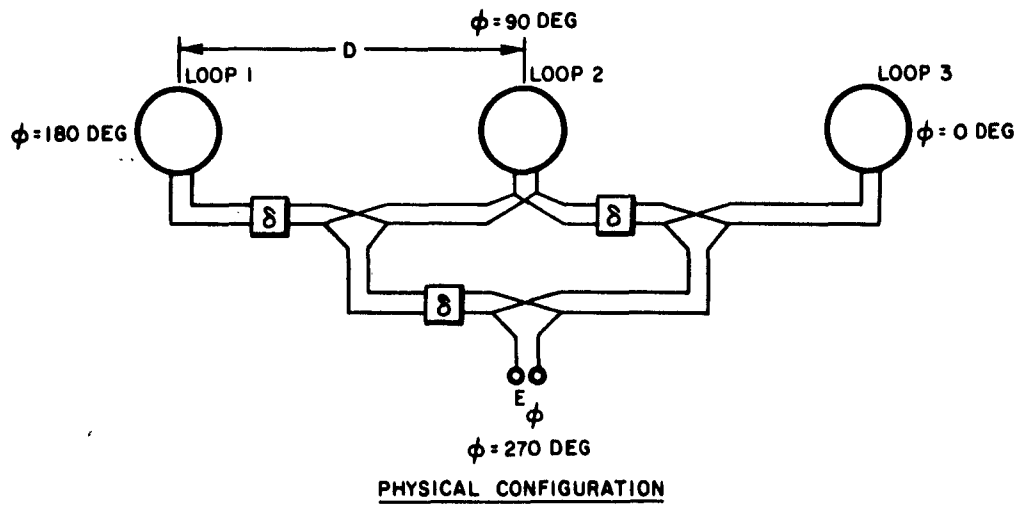
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PHYSICAL CONFIGURATION

PATTERN

FIGURE 1. Superdirective Array

The beam width is a measure of the directivity of an antenna. The half-power beam width, $2\varphi_A$, of the n-loop array in terms of the null position is

$$\cos \varphi_A = 0.707 \left(\frac{1 - \cos \varphi_0}{\cos \varphi_A - \cos \varphi_0} \right)^{n-1} \tag{5}$$

when $D \ll \lambda$. The narrowest beam width occurs when the null position approaches 90 deg. Then

$$2\varphi_A = 2 \text{ arc } \cos \sqrt[n]{0.707} \tag{6}$$

When the null position is at 180 deg, the beam width is greatest. The beam widths at these two extremes are plotted in Figure 2 as a function of the number of loops in the array. There is a spread of only about 10 deg between the extremes where the numbers of loops are greater. The beam width is not a function of the distance between loops, and herein lies the possibility of obtaining superdirectivity.

The amplitudes of the back lobes are an indication of the unidirectional properties of the array pattern. Although there are three back lobes, one at $\varphi = 180$ deg and two symmetrically located about this one (see Figure 1), only the lobe at $\varphi = 180$ deg is called the back lobe; the two lobes on either side of the back lobe are called side lobes. The ratio of the front lobe to the back lobe as derived from equation (3) is

$$R_0 = \left(\frac{1 - \cos \varphi_0}{1 + \cos \varphi_0} \right)^{n-1} \tag{7}$$

It is obvious from equation (7) that a large number of loops in the array would cause the back lobe to be much smaller than the front lobe. The front-lobe-to-side-lobe ratio can also be derived from equation (3). To do this, the position of the side lobe maximum must first be determined by differentiating equation (3) and equating the results to zero. Then the position of the side lobe is

$$\varphi_1 = \text{arc } \cos \left(\frac{1}{n} \cos \varphi_0 \right) \tag{8}$$

Equation (8) can now be used in conjunction with (3) to derive the front-lobe-to-side-lobe ratio, which is

$$R_1 = \left(\frac{n}{\cos \varphi_0} \right)^n \left(\frac{1 - \cos \varphi_0}{1 - n} \right)^{n-1} \tag{9}$$
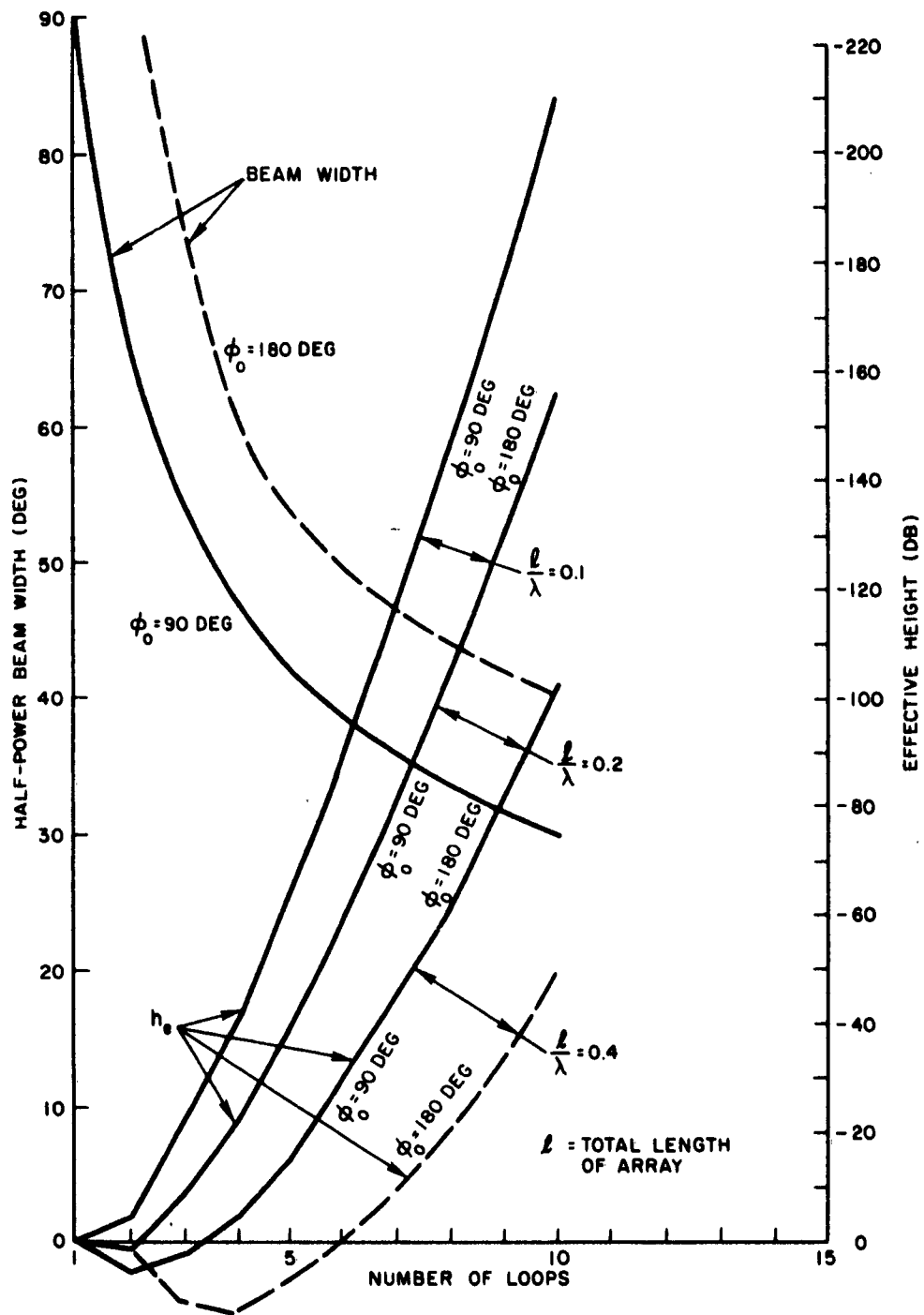
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FIGURE 2. Beam Widths and Effective Heights for
Superdirective Arrays

From equations (7) and (9), it is obvious that the loop array patterns become very unidirectional as the number of loops is increased. This is important in applications where the loop array is in a field of multiple sources, such as sferics at VLF.

EFFECTIVE HEIGHT

Moderately narrow beams can be achieved with a few loops, but in return, the effective height is reduced; this is shown in Figure 2. The effective height of an n-loop array as derived from equation (3) is

$$h_e = 20(n - 1)\log_{10}\left[\frac{2\pi D}{\lambda}(1 - \cos\varphi_0)\right] \qquad (10)$$

This is the effective height in db for an n-loop array compared with that for a one-loop array. In Figure 2, the effective height is plotted versus the number of loops for three different array lengths. The vertical distance between equal-array-length curves indicates the range of effective heights for null positions from 90 deg to 180 deg. The very small effective heights of the shorter arrays could make them impractical unless loops with very large effective heights are used to make up the array. Large ground-return inverted loops or perhaps short Beverage antennas would be a practical element to use because of their large effective heights.

THE EFFECT OF PHASE AND AMPLITUDE ERRORS

The foregoing analysis has assumed that the received signals have been of equal amplitude and that each loop voltage has had the proper phase to cancel at the null angles. As the number of loops is increased, any departure from this assumption will reduce the null depths. An analysis of the null voltage where there are small inequalities in loop-voltage phase and amplitude has been made (Ref. 9). The resultant null voltage was found to be the sum of the voltage amplitude differences between adjacent loops, $\Delta e_{n-1,n}$, in quadrature with the sum of the phase differences in radians between adjacent loops, $\Delta\theta_{n-1,n}$, which is expressed as

$$E_r = \left|\sum_2^n \Delta e_{n-1,n} - jE_L\sum_2^n \Delta\theta_{n-1,n}\right| \qquad (11)$$

where E_L is the amplitude of the voltage from one loop. The voltage differences between adjacent loops will tend to deteriorate the reception pattern as the number of loops is increased. Because highly directive loop arrays require accurate matching of individual loops and delay lines, the feasibility of a practical array will depend upon the degree of accuracy attainable, and this can only be determined by experiment.

SUMMARY AND CONCLUSIONS

Superdirectivity may be achieved with short VLF loop arrays because the beam width is not a function of the length of the array but of the number of loops in the array. Also, the usual limiting factors in superdirective arrays, such as narrow bandwidth and high losses, are extremely small because of the decoupling between loops at long wavelengths. The directivity is limited, however, by the critical tolerance in adjacent loop voltages.

The effective height and all of the pattern characteristics of the short array, such as beam width and back-lobe amplitude and position, can be expressed in terms of the selected null position and the distance between loops for a given number of loops. The assumption is made that the distance between loops is much smaller than a wavelength, which is valid at VLF. The directivity is increased by the number of loops used in the array. Equations (6), (7), and (9) bear this out in that the beam width and back lobes become smaller as the number of loops increases.

The two limiting factors that affect the directivity are effective height and unequal voltages between loops. The very small effective heights of the shorter arrays could make them impractical unless very large loops are used. The most serious limitation on the directivity of an array with a large number of loops is caused by the voltage-amplitude and phase differences between loops. These differences between adjacent loops will obscure the nulls and deteriorate the reception pattern. Equation (11) shows that the differences add together as the number of loops is increased. The feasibility of designing highly directive loop arrays will depend on the accuracy with which the individual loops and delay lines can be matched; this must be experimentally determined.

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